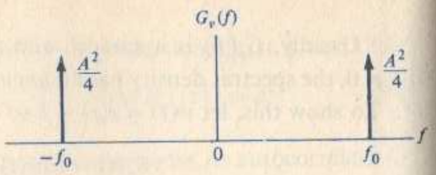

FIGURE 3.8
Power spectral density of a randomly phased sinusoid.

which is identical to the autocorrelation of a *deterministic* sinusoid—see Example 2.15, Sect. 2.6.

Fourier transformation then gives an impulsive power spectral density

$$G_v(f) = \frac{A^2}{4}\,\delta(f - f_0) + \frac{A^2}{4}\,\delta(f + f_0) \qquad (10b)$$

as sketched in Fig. 3.8. We see that this random signal has specific amounts of average power concentrated at $f = \pm f_0$ and no spectral density elsewhere, in agreement with intuitive reasoning. Moreover, $\int_{-\infty}^{\infty} G_v(f)\,df = A^2/2$ which clearly equals $\overline{v^2}$, and the fact that there is no impulse at $f = 0$ agrees with $\bar{v} = 0$. ////

Example 3.7 Random Binary Wave ★

The previous example is rather deceptive owing to its simplicity. More often than not, determining an autocorrelation function from scratch is a sticky analytic problem—as this example will demonstrate.

Figure 3.9 is a sample function of the random binary wave $v(t)$. During any time interval $(n - 1)T < t - T_d < nT$, $v(t)$ takes on either the value $+A$ or $-A$. These two amplitudes are equally likely, and the amplitude in any one interval is independent of all other intervals. The delay term T_d is a random variable over the ensemble, uniformly distributed over $[0,T]$.

To find the autocorrelation, we first consider $|\tau| > T$; so, for any sample function, $v(t)$ and $v(t - \tau)$ are in different intervals and hence are *independent*. Thus

$$\mathrm{E}[v(t)v(t - \tau)] = \bar{v}^2 = \langle v(t)\rangle^2 = 0$$

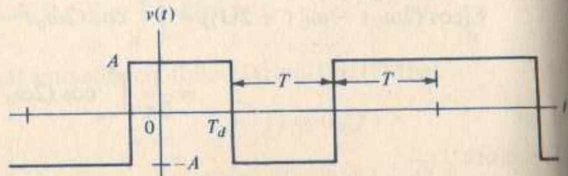

FIGURE 3.9
Sample function of a random binary wave.

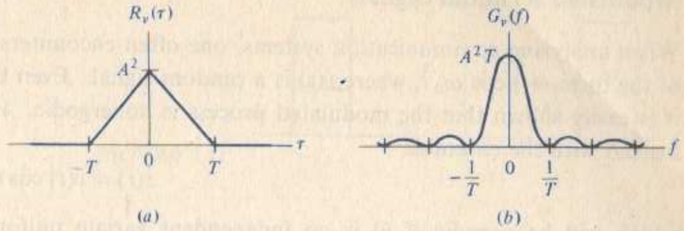

FIGURE 3.10
Random binary wave. (*a*) Autocorrelation; (*b*) power spectral density.

where the time average $\langle v(t)\rangle = 0$ follows by inspection of $v(t)$.

Now take $|\tau| < T$ and let $t = 0$; then $v(0)$ and $v(-\tau)$ are in the same interval only if $T_d - T < -|\tau|$. Thus

$$\mathrm{E}[v(0)v(-\tau)] = \begin{cases} A^2 & T_d < T - |\tau| \\ 0 & \text{otherwise} \end{cases}$$

The probability that $T_d < T - |\tau|$ is

$$P(T_d < T - |\tau|) = \int_0^{T-|\tau|} \frac{dt_d}{T} = \frac{T - |\tau|}{T}$$

so

$$\mathrm{E}[v(0)v(-\tau)] = A^2 P(T_d < T - |\tau|) = A^2\left(1 - \frac{|\tau|}{T}\right)$$

By like reasoning for any value of t

$$R_v(\tau) = \mathrm{E}[v(t)v(t - \tau)] = \begin{cases} A^2\left(1 - \dfrac{|\tau|}{T}\right) & |\tau| < T \\ 0 & |\tau| > T \end{cases}$$

$$= A^2 \Lambda\left(\frac{\tau}{T}\right)$$

where $\Lambda(\tau/T)$ is the triangle function. Finally, drawing upon the known transform pair,

$$G_v(f) = A^2 T \operatorname{sinc}^2 fT$$

Figure 3.10 shows the autocorrelation function and power spectrum of the random binary wave. ////

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To the memory of my father,
ALBIN JOHN CARLSON